UKR Journal of Multidisciplinary Studies (UKRJMS)
Homepage: https://ukrpublisher.com/ukrjms/
Email: submit.ukrpublisher@gmail.com
Volume 2, Issue 2, 2026
ISSN: 3049-432X (Online)

Promotion of Some Mathematical Research Fields and Physical Applications

Yi-Fang Chang

Department of Physics, Yunnan University, Kunming, 650091, China

***Corresponding Author**: Yi-Fang Chang

DOI: *https://doi.org/10.5281/zenodo.18730504*

Article History

Original Research Article
Received: 25-01-2026
Accepted: 13-02-2026
Published: 22-02-2026

Citation: Yi-Fang Chang. (2026). Promotion of some mathematical research fields and physical applications. UKR Journal of Multidisciplinary Studies (UKRJMS), Volume 2(2). 117-124.

Abstract

First, integer rings extend to fraction and general number fields in number theory, which obtains Fermat's Last Theorem has no integer and fraction solution, and so on. Second, we research some promotions in mathematical fields. Third, when the series of positive terms are extended to the positive continuous function, there have the integral test and differential test. Fourth, we discuss renormalization group in mathematics, and some extensive theories and applications in physics, and negative matter.

***Key words**: promotion, integer ring, field, number theory, test, extension, renormalization, application.*

1. Introduction

It is well-known scientific developed history that mathematics and physics promote each other. The continuous generalization of number fields in mathematics is a great advance, such as the complex variable functions. Two of the biggest developments in science in the 20th century are related to this, the time of the 4-dimensional space-time in relativity is imaginary, and the operator and wave equations in quantum mechanics are generalized to complex numbers. In Feynman quantum mechanics the path integral is the complex number, whose modulus square gives the probability.

In advanced mathematics, the conventional three-dimensional framework of geometry and calculus has been systematically generalized to higher-dimensional (n-dimensional) spaces, including abstract structures such as Hilbert spaces [1]. Within mathematics and theoretical physics, geometric field theory constitutes a central domain of inquiry. When calculus is extended to higher dimensions—encompassing fractal geometries, complex spaces, and generalized number systems—field-theoretic formulations and associated analytical tools must likewise be reformulated. In such generalized settings, classical integral theorems, including Gauss's theorem and Stokes's theorem, together with their extensions involving gradient, divergence, and curl operators, require appropriate higher-dimensional adaptations [2].

The extension of geometric dimension to n-dimensional manifolds naturally broadens the concept of fields from scalar and vector representations to more sophisticated mathematical entities such as tensors, spinors, torsion fields, and twistors. These generalized structures find applications across diverse scientific disciplines, including physics, biology, geophysics, and the social sciences. Field theory has thus become an interdisciplinary framework whose theoretical refinements often stimulate new applications in both natural and social domains [3].

For two any sets M and N, we researched the simple mathematical joint relation of algebra in set theory $(M,k)(k',N)=(M,kk',N)$, and discussed its some properties. It may describe various relations among sciences and many intersecting sciences. This is applied to various aspects in natural science and social science, which includes transcription, splicing and replication, etc., in biology, and both main relations in micro-economics and macro-economics, etc. Further, it may be corrected and developed [4].

Following the Einstein–Podolsky–Rosen (EPR) prediction, quantum nonlocality and entanglement have become central themes in contemporary physics. We briefly review selected investigations of EPR phenomena, including contributions by Santilli. Within a framework grounded in a generalized Lorentz transformation (GLT) that incorporates superluminal extensions of special relativity, we propose that entangled states should satisfy GLT relations, given their superluminal features and spacelike characteristics. Under this formulation, modifications arise in the phase structure of the entangled field, leading to the introduction of a hypothetical phase excitation, or "phason," governed by corresponding dynamical equations. This excitation is conceptually associated with tachyonic behavior and is assumed to resemble a photon with spin J=1 and either zero or extremely small mass, analogous to a neutrino, potentially accounting for action-at-a-distance effects. Treating the entangled field as a wave entity, we explore its characteristic properties and examine the theoretical possibility of superluminal quantum communication via correlated entangled states or paired instruments prepared and transmitted across distinct locations. In such a scenario, manipulation at one site could correspond to information transfer to the other. We further speculate that the entangled field may admit an analogy with magnetic field theory, potentially representing a quantum cosmic field or an extension of quantum theory, conceptually related to the nonlinear hidden-variable framework of de Broglie–Bohm theory. Overall, the investigation of nonlocality and entangled fields carries significant theoretical and interdisciplinary implications [5], and this paper outlines prospective developments within both mathematical and physical domains.

2. Integer Rings Extend to Fraction and General Number Fields in Number Theory

It is known that general integer numbers are divided into prime p and combination number np, and unit number 1.

Fermat's Last Theorem is:

$$a^n + b^n = c^n. \qquad (1)$$

For any positive integer n greater than 2, equation (1) has no integer solution on a ring of positive integer. In 1955 Taniyama-Shimura conjecture is proposed. In 1984 G. Frey proposed Eq.(1) may change an elliptic equation:

$$y^2 = x^3 + (a^n - b^n)x^2 - a^n b^n. \qquad (2)$$

After 358 years of continuous efforts, finally in 1994 Andrew Wiles proved that there is no solution for integers [6].

For fractions we can prove simply:

$$(\frac{a}{a_0})^n + (\frac{b}{b_0})^n = (\frac{c}{c_0})^n, \qquad (3)$$

which is also no solution.

Proof: Fermat's Last Theorem extends to fractions Eq. (3):

$$(\frac{a}{a_0})^n + (\frac{b}{b_0})^n = \frac{b_0^n a^n + a_0^n b^n}{a_0^n b_0^n} = \frac{c^n}{c_0^n}, \qquad (4)$$

i.e. $(ab_0c_0)^n + (a_0bc_0)^n = (a_0b_0c)^n$. (5)

Such equations (5) and any rational number (3) are also no solution [6]. A special example is c=1.

But, for irrational number and real number $c = (a^n + b^n)^{1/n}$ must hold. For complex numbers, especially Gauss integers $m + in$ (m and n are all integers, for polynomials, m-n symmetry may be generally true), should also hold.

Eq. (1) for n=2 hold, which may be any basic solution:

$$a = m^2 - n^2,\ b=2mn,\ c = m^2 + n^2. \qquad (6)$$

Eq. (5) provides other relations.

Fermat-Pell equation is [7,8]:

$$x^2 - Ny^2 = 1. \qquad (7)$$

If α, β are solutions, α^{-1}, $\alpha\beta$ and α^k (k may be positive numbers, negative numbers, or zero) are all solutions.

Eq.(7) extends to fractions:

$$(\frac{x}{x_0})^2 - N(\frac{y}{y_0})^2 = 1, \qquad (8)$$

i.e. $(xy_0)^2 - N(x_0y)^2 = (x_0y_0)^2$. (9)

Further [9],

$$x^n + y^n = n! z^n, \qquad (10)$$

and $x^n \pm y^n = n!$. (11)

Both are no integer solution. By the same method, for fractions

$$(\frac{x}{x_0})^n + (\frac{y}{y_0})^n = n!(\frac{z}{z_0})^n, \qquad (12)$$

i.e. $(xy_0z_0)^n + (x_0yz_0)^n = n!(x_0y_0z)^n$. (13)

We prove that Eq.(12) is also no solution. A special example z=1, which is similar with Eq.(11).

Prime with $x^2 + 1$ form should have infinite [7]. It extends to fractions $(\frac{x}{x_0})^2 + 1$, i.e. $x^2 + x_0^2$.

In number theory, this is integer ring extended to various number rings and number fields, and to rings of polynomial functions O_C.

It may be similar generalization to various other theorems in number theory. Such as the congruence [A=rm+B, i.e., A=B(mod m)]. The concept of congruence has been generalized to fractions [7].

Prime twins n-1 and n+1 may be n=4, 6, 12, 18, 30, 42, 60, 72, 102, etc. If x from 1 to 20, the formula n=30(2x-27)(x-15) may obtain the prime twins.

Prime twins develop to fractions $\frac{p}{a}-1$ and $\frac{p}{a}+1$. The difference between any prime p(>3) must be an even number 2*a*. *a*=1 is namely prime twins, and should hold; but, it will tend to be less and less. Most likely prime twins are 10n+1 and 10n+3. Such as (11-13), (41-43), (71-73), (101-103), (191-193), (641-643), etc.

Let $\pi(x)$ is the number of prime less than or equal to x, and its approximate formulas have [7] Tchebychefff-Gauss formula:

$$\pi(x)=\int_2^x \frac{1}{\ln t}dt. \quad (14)$$

Legendre formula:

$$\pi(x)=\frac{x}{\ln x-1.08366}. \quad (15)$$

If x>400000, so there has [8]:

$$\frac{1}{3}\frac{x}{\log x}\le \pi(x)\le \frac{10}{3}\frac{x}{\log x}. \quad (16)$$

Fermat theorem $x^{p-1}=1$ (mod *p*). Wilson theorem $(p-1)!=-1$ (mod *p*).

$p=4x+1=m^2+n^2$. Here m and n must be one odd and one even.

$x^4-y^4=z^2$ is no solution.

For n, the sum of all positive factors except itself is $\sigma(n)-n$, so the necessary and sufficient condition for the perfect number is $\sigma(n)=2n$. But, the perfect numbers are all even, and whether there are infinite many. Both are without not proof.

Goldbach's conjecture is $2n=p_1+p_2$.

3. Some Promotions in Mathematics

Various mathematical formulas of equal powers:

$$f(x_1,x_2,...x_i)=\sum \pm(x_1^{n_1}x_2^{n_2}...x_i^{n_i})=0 ,$$

$$(n_1+n_2+...+n_i\equiv m). \quad (17)$$

Either holds or is not hold for integer and fraction-rational numbers. For instance, the formula of equal powers:

$$\sum a_i x_i^n=\sum b_j y_j^n. \quad (18)$$

When $x_i\Rightarrow \frac{x_i}{x_{0i}}, y_j\Rightarrow \frac{y_j}{y_{0j}}$, Eq.(18) becomes the same form:

$$\sum a_i(x_i y_{0j})^n=\sum b_j(x_{0i}y_j)^n. \quad (19)$$

A special case is the quadratic homogeneous formula. Its general form is:

$$f(x_1,x_2,..x_n)=\sum_{i=1}^n\sum_{j=1}^n a_{ij}x_ix_j. \quad (20)$$

Pass through the quadratic homogeneous matrix form:

$$f(x_1,x_2,...x_n)=X'AX. \quad (21)$$

This can be transformed into the standard form of quadratic homogeneity:

$$f(x_1,x_2,..x_n)=\sum_{i=1}^r \lambda_r y_i^2. \quad (22)$$

This corresponds to either an elliptic equation or a hyperbolic equation:

$$(\frac{x}{a})^2+(\frac{y}{b})^2\pm(\frac{z}{c})^2=\pm 1, \quad (23)$$

and the classification of the general quadratic equation.

This relates that in 1922 the British mathematician Model proposed a famous conjecture: Model conjecture. In its original form, any binary polynomial with an irreducible rational coefficient has a finite solution when its genus than or equal to 2. This polynomial is f(x,y), which implies that at most there are finite number-even pairs x_i, y_i ∈Q, and $f(x_i,y_i)=0$. Then this conjecture extends to polynomials defined in arbitrary number fields, and is revisited with the emergence of abstract algebraic geometry, and with algebraic curves.

In 1929 Siegel proposed the integral point theorem on curves, and later K. Mahler extends to integer solutions and rational number solutions. Further, in 1975 Robinson and Roquette applied the nonstandard analysis [10] for nonstandard integral points and nonstandard prime factors to generalize the Siegel-Mahler theorem [11].

Riemann zeta-function $\varsigma(s)=\sum_n \frac{1}{n^s}$ is related with prime distribution, and s may be the complex variable, and is related with the complex analysis. Riemann zeta-function may extend to Dedekind function.

Let $x=1+\sum_{n=1}^{m}2^n$, so $2x+1=1+2\sum_{n=1}^{m}2^n=1+\sum_{n=1}^{m+1}2^n$. If n is finite, it will add 2^{m+1}; if n is infinite, so 2x+1=x, $x=-1$. Similarly, it may derive $\sum_{n=1}^{\infty}n=-\frac{1}{12}$, and the dimension of superstring is 26 based on the infinite number of bosons.

4. New Differential Test for Series of Positive Terms and Its Completeness

In mathematical analysis, numerous criteria have been developed to determine the convergence or divergence of infinite series with positive terms [12]. In addition to the fundamental comparison test, well-established methods include the D'Alembert ratio test, the Cauchy root test, the Raabe test, the Kummer test, and the Abel–Dini test, among others [13,14]. When such series are associated with positive continuous functions, the integral test provides a corresponding analytical framework for assessing convergence. By employing an analogous methodological approach, we derive a new differential criterion applicable to series of positive terms and illustrate its validity through selected examples [15–18].

The differential test is: Let $\sum_{k=1}^{\infty}f(k)$ be a series of positive terms, *f(x)* is a corresponding positive continuous function, and *g(x)* is a derivative of reciprocal of *f(x)*, i.e.,

$$\frac{d}{dx}[\frac{1}{f(x)}]=g(x). \tag{24}$$

Then, if $fgx\geq 1+\alpha(\alpha>0)$ for enough large x, the series converges; if $fgx\leq 1$ the series diverges.

In calculus, differential is a simple operation that can be applied to composite expressions constructed from elementary functions; consequently, the proposed criterion retains structural simplicity and admits broad applicability. Furthermore, for any given series under consideration, the resulting inequality may be expressed in the form $fgx\geq 1+\alpha(\alpha>0)$ or $fgx\leq 1$, thereby providing a decisive condition for convergence or divergence. In this sense, the test may be regarded as both general in scope and theoretically comprehensive [16–18].

If *f* is a discrete function a_n, the difference is substituted for differentiation $g=\frac{a_{n+1}^{-1}-a_n^{-1}}{1}=\frac{a_n-a_{n+1}}{a_n a_{n+1}}$, $\lim_{n\to\infty}nfg=\lim_{n\to\infty}n(\frac{a_n}{a_{n+1}}-1)=c$, i.e., the Raabe test.

In many cases, the test may make the limit form, i.e., $fgx \to c$ as $x\to\infty$. Then the series converges if *c*>1 and diverges if *c*<1. Therefore, it may combine the L'Hopital's rule. Of course, it cannot test for c=1, since an infinitesimal α is neglected in the limit form. But, we use a general differential test to calculate any series, its results must be $fgx=1+\alpha$, in which α >0 is converges of series; or $\alpha\leq 0$ is divergence of series. For enough large x, α must be 0, or positive or negative infinitesimal, which is the hyperreal number of zero region in the nonstandard analysis [11]. Such when we combine the nonstandard analysis [17], the differential test must be universal and completeness [16-18].

Of course, the test of the limit form fails for c=1, since an infinitesimal α is neglected. But, we use a general differential test, the result must be $fgx\geq 1+\alpha(\alpha>0)$ or $fgx\leq 1$, i.e., be $fgx=1+\alpha$, in which α >0

Further, the differential test may be applied to test for the general series of functional terms $\sum_{n=1}^{\infty}f_n(x)$. This corresponding positive continuous function is ***f(x,y)***, and

$$g(x,y)=\frac{d}{dy}[\frac{1}{f(x,y)}]. \tag{25}$$

So $fgx\geq 1+\alpha(\alpha>0)$ or $fgx\leq 1$ determines the convergence domain of series. It may be applied to test for the infinite integral whose limits of integral from positive to infinite. The convergence or divergence of the integral may be determined by the differential of the integrand. If $fgx\geq 1+\alpha(\alpha>0)$ or $\lim_{x\to\infty}fgx=c>1$, so $\int_a^{\infty}f(x)dx,(a\geq 1)$ converges; $fgx\leq 1$ or c<1, $\int_a^{\infty}f(x)dx$ diverges.

5. Some Applications in Physics

5.1. Quantum Theory

Classical mechanics and electrodynamics are all study within the continuous real number field. Quantization, however, is the inverse process, transitioning from the real number field to the integer field. This includes the evolution of the Hall effect into fractional Hall effects, among other phenomena.

In quantum mechanics the operator representation of physical quantities is wave mechanics, and its matrix representation is matrix mechanics.

5.2. Renormalization Group and Various Extensive Theories

More generally it corresponds to some scales of amplification or reduction, and corresponds to the renormalization and the renormalization group.

Renormalization theory is origin of the quantum field [19,20], and is applied to many branches in physics [21,22]. The renormalization group is a discrete semigroup, and may have Hopf algebra structure [23].

This is mathematical base of various extensive theories. Their physics correspond to the extensive quantum theory [24-27], the extensive special relativity and general relativity [28-30], and various extensive theories [31].

For the extensive general relativity [30] it may be based on Riemannian geometry and the extensive Lobachevsky geometry, etc., combined various curved space-time; the electromagnetic general relativity [30]; other interactions, for example, strong and weak interactions [32,31], the thought field [33-35], etc.

The non-Abel gauge field equations are:

$$D_{\mu}F^{\alpha}_{\mu\nu} = -J^{\alpha}_{\nu}\,. \tag{26}$$

From a purely geometric point of view, this means that the geometric properties of the fiber space will be determined by the matter field, and vary according to its dynamics. This is a promotion of general relativity.

The framework of extensive quantum theory may be constructed upon a generalized wave-theoretic foundation, incorporating elements such as a reformulated Titius–Bode relation for planetary spacing in the solar system, diverse quantization schemes, and principles derived from discrete mathematics. One significant application of this theoretical extension lies in extensive quantum biology [36–39], within which proposed solutions have been associated with structural features such as the double-helical configuration of DNA [38,39]. Beyond the natural sciences, the theory has been further extended to formulate concepts of social extensive electrodynamics and a generalized relativistic model [40], as well as exploratory applications within theological contexts [41], thereby suggesting a broad interdisciplinary scope for its continued development.

Any field quantity $\chi(x)$ is under the scaling transform x→x'=x/k, if the transformation defines as:

$$\chi(x) \to \chi'(x) = k^{d}\chi(kx)\,. \tag{27}$$

A general equation of renormalization group is:

$$(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + \gamma_m m\frac{\partial}{\partial m} + \gamma_{\Gamma})\Gamma_n(p_j, g, m, \mu) = 0\,. \tag{28}$$

The scalar field equation is:

$$(\partial_{\mu} - igA^{\alpha}_{\mu}t^{\alpha})^2\varphi - m^2\varphi = 0\,. \tag{29}$$

The spinor field equations are:

$$\gamma_{\mu}(\partial_{\mu} - igA^{\alpha}_{\mu}T^{\alpha})\psi + M\psi = 0\,. \tag{30}$$

For the extensive string [32], membrane and various important theories, these theories representations are the same with those original theory, only the basic constants can be different. The scaling invariance is mathematical basis of various extensive theories.

5.3. Negative Matter

Dark matter and dark energy have become key topics in modern astronomy, astrophysics, cosmology, and fundamental physics, attracting ongoing global scientific attention. Nonetheless, empirical validation of numerous theoretical models continues to be technically difficult because of observational constraints and the indirect character of the evidence at hand. Einstein questioned why gravitational masses universally exhibit the same sign as early as 1954, pointing out a conceptual imbalance in classical gravitational theory. Since 2007, we have developed a paradigm that expands the concept of mass to encompass a negative counterpart, building upon this underlying issue [42–56].

Utilizing Dirac's notion of negative energy states, alongside Einstein's mass–energy equivalence and the equivalence principle—which posits the equivalence of inertial and gravitational mass—we propose a negative matter model as a fundamental and cohesive theoretical framework for both dark matter and dark energy. In this formulation, the initial statement characterizing negative matter inside Newtonian gravitation is presented as follows:

$$F = -\frac{G}{r^2}M_1M_2\,. \tag{31}$$

In this concept, the distinguishing characteristic of negative matter is its mutual gravitational attraction with matter of the same sign, while producing universal gravitational repulsion in interactions with positive matter. As a result, positive and negative matter would inhabit different, topologically distinct regions, with negative matter appearing as dark matter due to its invisibility and as dark energy through its repulsive gravitational influences. The model is shown to be in line with Occam's Razor because it tries to explain both dark matter and dark energy phenomena in a single, simple way.

When looking at systems made up of both positive and negative mass, there are only three basic ways for them to interact. Based on this, we say that Bondi's formulation has problems within itself. A thorough analysis of current demonstrations of the positive mass (energy) theorem reveals that these proofs depend on particular foundational assumptions, so constraining their validity. Additionally, it is imperative to meticulously differentiate between antimatter and negative matter, as these notions are both physically and theoretically separate.

The primary conclusion of this theory is that known physical frameworks—spanning classical mechanics, relativity, and quantum theory—can be preserved in their formal structure, with the main alteration being the extension of mass to encompass both positive and negative values. In the context of general relativity, the phrase for negative matter is articulated as follows:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi k (T_{\mu\nu} - T'_{\mu\nu}) . \quad (32)$$

Here the cosmological constant Λ corresponds to the negative matter $\Lambda = 8\pi k T'_{\mu\nu} / g_{\mu\nu}$. From the total energy formulation, it can be inferred that the rotating motion of galaxies remains about constant, while the ratio of total matter to ordinary (baryonic) matter transitions from unity in the early universe to current estimates of approximately 11.82 or 7.88. In this model, the start of faster cosmic expansion is expected to happen around 9.76 billion years. The model also suggests a way for cosmic inflation to happen. It says that positive and negative matters come out of a vacuum state at the same time, and that strong interactions at very small scales cause exponential expansion. Furthermore, by examining gravitationally repulsive lensing phenomena linked to negative matter, quantifiable observational methodologies are proposed for the detection of negative dark matter within the Milky Way and other astronomical systems. In this framework, three distinct classifications of dark regions in the universe are delineated: massive black holes, low-mass nebular structures, and negative dark matter. Each category exhibits unique physical properties and can, in theory, be differentiated through observational data from contemporary astronomical instruments.

From this theoretical standpoint, "phantom" energy can be viewed as a representation of negative matter, while anti-gravity is characterized as the repulsive force between negative and positive mass components. In conclusion, although dark matter and dark energy are frequently perceived as mysterious, they do not have to remain conceptually ambiguous. Negative matter is suggested as a singular entity that can explain both occurrences within a straightforward and internally consistent framework that allows for quantitative analysis and possible empirical validation. If observable evidence ultimately substantiates the existence of negative matter, it would signify a significant enhancement of our comprehension of fundamental physics and cosmology.

6. Summary

Space-time (t, x) is first quantized, and developed into high dimension, fractal dimension and complex number space-time. Various constants (c, h, G), etc., are also developed.

Further mass should develop into complex mass M→M+im, which corresponds to the complex time. The complex number momentum corresponds to the spatial complex number.

The system of natural units (c=h=1) is namely a unified extensive form. This should have the corresponding renormalization group equation, in the original equation, μ and m transform to c, h, G, etc. It may be based on the system theory, symmetry, fractal, scaling, etc.

7. References

1. R.Courant and F.John. *Introduction to Calculus and Analysis*. John Wiley and Sons, Inc. **1982**.
2. Yi-Fang Chang, Some new researches of field theory and their applications. *International Journal of Physics and Applications*. 2021,3(1): 25-30.
3. Yi-Fang Chang, Some developments and applications on field theory in geometry. Algebras, Groups, and Geometries. *Algebras, Groups, and Geometries*. 2023, 39(1):113-126.
4. Yi-Fang Chang, Simple mathematical joint relation and its applications in intersecting sciences, biology and social science, etc. *Algebras, Groups, and Geometries*. 2017, 34(4): 425-443.
5. Yi-Fang Chang, Nonlocality, entangled field and its predictions, superluminal communication. Algebras, Groups, and Geometries. 2019-2020, 36(2):481-494.
6. Andrew Wiles, Modular elliptic curves and Fermat's Last Theorem. Annals of Mathematics. 1995, 141(3): 443-551.
7. Albert Beiler, Recreations in the Theory of Numbers. Dover Publications, Inc. 1964.
8. U. Dudley, Elementary Number Theory (Second Edition). Dover Publications. 2008.
9. R.K. Guy, Unsolved Problems in Number Theory. Springer Verlag. 1994.
10. A. Robinson, *Non-Standard Analysis*. North-

Holland Publishing Company. 1974.

11. A. Robinson and P. Roquette, Journal of Number Theory. 1975.

12. L.H. Loomis, *Calculus* (Third Edition). Addison-Wesley Publishing Company. 1982.

13. G.H. Hardy, *A Course of Pure Mathematics* (Tenth Edition). Cambridge Mathematical Library. 1993.

14. T.J.I'A. Bromwich, *An Introduction to the Theory of Infinite Series*. AMS Chelsea Publishing. 2005.

15. Yi-Fang Chang, *New Research of Particle Physics and Relativity.* Yunnan Science and Technology Press. **1989**. p244-246.

16. Yi-Fang Chang, Differential test for series of positive terms. J.Yunnan Normal University. 1999, 19(3):5-7.

17. Yi-Fang Chang, Differential test for series of positive terms, new tree-field representations in graph theory and new number field, extension of Dirac extraction, and their applications. International Journal of Sciences. 2012,1,Dec. 121-133.

18. Yi-Fang Chang, New tree-field representations in graph theory, extension of Dirac extraction, differential test for series of positive terms, complex dimension and their applications. International Journal of Modern Mathematical Sciences. 2014,9(1):1-12.

19. J.D. Bjorken, S.D. Drell, Relativistic Quantum Mechanics. McGraw-Hill Book Company. 1964.

20. J.D. Bjorken, S.D. Drell, Relativistic Quantum Fields. McGraw-Hill Book Company. 1965.

21. K.G. Wilson, Renormalization group and critical phenomena. Rev.Mod.Phys. 1983,55,583-599.

22. G. Benfatto, G. Gallavotti, Renormalization Group. Princeton University Press. 1995.

23. A. Connes, D. Kreimer, Hopf algebra, renormalization and noncommutative geometry. Comm. Math. Phys. 1998,199,203.

24. Yi-Fang Chang, Development of Titius-Bode law and the extensive quantum theory. Physics Essays. 2002,15(2):133-137.

25. Yi-Fang Chang, Nanophysics, macroscopic quantum phenomena and extensive quantum theory. International Journal of Nano and Material Sciences. 2013,2(1):9-24.

26. Yi-Fang Chang, Extensive quantum theory with different quantum constants, and its applications. International Journal of Modern Mathematical Sciences. 2018,16(2):148-164.

27. Yi-Fang Chang, Restructure of quantum mechanics by duality, the extensive quantum theory and applications. Physical Science & Biophysics Journal. 2024,8(1):000265.1-9.

28. Yi-Fang Chang, New Research of Particle Physics and Relativity. Yunnan Science Technology Press. 1989. Phys.Abst. 93,1371, 30443(1990).

29. Yi-Fang Chang, Extension and complete structure of the special relativity included superluminal and neutrino-photon with mass. International Journal of Modern Theoretical Physics. 2013,2(2):53-73.

30. Yi-Fang Chang, GRT extended for electromagnetic fields: equivalence principle and geometrization. Galilean Electrodynamics. 2005,16(5):91-96.

31. Yi-Fang Chang, Extended superstring and unification of four interactions. Hadronic Journal. 2023,46(1): 97-117.

32. Yi-Fang Chang, Unification of strong-weak interactions and possible unified scheme of four-interactions. European Journal of Applied Sciences. 2020, 8(5):28-45.

33. Yi-Fang Chang, Living beings and the thought field. The Journal of Religion and Psychical Research. 2003,26(2):98-102.

34. Yi-Fang Chang, The thought field, the thermodynamics of physiology and psychology and decrease of entropy. The Journal of Spirituality and Paranormal Studies. 2006,29(2): 62-68.

35. Yi-Fang Chang, Four possible mathematical-physical models of "soul" and the thought field. World Institute for Scientific Exploration (WISE) Journal. 2018,7(2):84-92.

36. Yi-Fang Chang, Extensive quantum biology, applications of nonlinear biology and nonlinear mechanism of memory. NeuroQuantology. 2012, 10(2):183-189.

37. Yi-Fang Chang, Extensive quantum theory of DNA and biological string. NeuroQuantology. 2014,12(3):356-363.

38. Yi-Fang Chang, Some solutions of extensive quantum equations in biology, formation of DNA and neurobiological entanglement.

NeuroQuantology. 2015,13(3): 304-309.

39. Yi-Fang Chang, The double helical structure of DNA in quantum mechanics, and nonlinear biomechanics. Physical Science & Biophysics Journal. 2022,6(2):000223.1-9.

40. Yi-Fang Chang, Causality, social extensive electrodynamics and general relativity, and orbits of life and society. International Journal of Modern Social Sciences. 2014,3(3):201-212.

41. Yi-Fang Chang, Three dimensional body-mind-spirit worlds in human society, and extensive quantum theory in theology. American Journal of Humanities and Social Sciences Research (AJHSSR). 2024, 8(7): 119-128.

42. Yi-Fang Chang, Negative matter, repulsion force, dark matter and inflation cosmos, Higgs mechanism. 2007. arXiv:0705.2908.

43. Yi-Fang Chang, Negative matter, dark matter and theoretical test. *International Review of Physics*. 2011,5(6): 340-345.

44. Yi-Fang Chang, Field equations of repulsive force between positive-negative matter, inflation cosmos and many worlds. *International Journal of Modern Theoretical Physics*. 2013, 2(2):100-117.

45. Yi-Fang Chang, Astronomy, black hole and cosmology on negative matter, and qualitative analysis theory. International Journal of Modern Applied Physics. 2014, 4(2):69-82.

46. Yi-Fang Chang, Optics in cosmos: dark matter, negative matter, GRB, and some problems on light. International Journal of Advanced Research in Physical Science (IJARPS). 2014, 1(5):11-20.

47. Yi-Fang Chang, Negative matter as unified dark matter and dark energy, and possible tests. Hadronic Journal. 2017, 40(3):291-308.

48. Yi-Fang Chang, Negative matter as dark matter, and its judgment test and calculation of ratio. International Journal of Modern Applied Physics. 2019, 9(1):1-12.

49. Yi-Fang Chang, Negative matter as unified dark matter and dark energy: simplest model, theory and nine tests. *International Journal of Fundamental Physical Sciences*. 2020, 10(4):40-54.

50. Yi-Fang Chang, Development of matter and testable negative matter as unified dark matter and dark energy. *Philosophy Study*. 2021, 11(7): 517-526.

51. Yi-Fang Chang, The premises and simple estimation on proof of the positive mass theorem, and negative matter as unified dark matter and dark energy. Journal of Pure and Applied Mathematics. 2022, 6(6):15-21.

52. Yi-Fang Chang, Basis on negative matter as unified dark matter and dark energy. SCIREA Journal of Astronomy. 2023, 5(1):1-11.

53. Yi-Fang Chang, Possible distributions of negative-dark matter in various galaxies, and new research for some peculiar galaxies. European Journal of Theoretical and Applied Sciences. 2024, 2(1):458-467.

54. Yi-Fang Chang, Negative matter of unified dark matter and dark energy and its observed ways in the Milky Way, and possible mechanism of missing antimatter. International Journal of Modern Applied Physics. 2024,13(1):11-25.

55. Yi-Fang Chang, Symmetric nature of positive-negative matters, and its generalization and test. Physical Science & Biophysics Journal. 2025,9(1):000281.1-10.

56. Yi-Fang Chang, Negative Matter as Unified Dark Matter and Dark Energy: Theories and Possible Observations. Sara Book Publication. 2025.